

Database Systems
Spring 2026
Suggested Solution for Homework #7

[1] See h7q1_sol.pdf.

[2] $F = \{A \rightarrow B, AB \rightarrow C, CD \rightarrow E, E \rightarrow F\} \mid - AD \rightarrow F$

Proof.

[1] $A \rightarrow B$ (given)

[2] $AB \rightarrow C$ (given)

[3] $A \rightarrow C$ (pseudo-transitivity rule on [1] and [2] and simplification of 2 occurrences of A to 1.)

[4] $CD \rightarrow E$ (given)

[5] $AD \rightarrow E$ (pseudo-transitivity rule on [3] and [4])

[6] $E \rightarrow F$ (given)

[7] $AD \rightarrow F$ (transitivity axiom on [5] and [6])

[3]

[a] $R(A,B,C,D) \{B \rightarrow C, C \rightarrow DA\}$

CK: [1] B

Highest NF: 2NF

Reason: $C \rightarrow DA$ violates 3NF since C is not a superkey, and D and A are non-prime.

Canonical Cover (optional): $\{B \rightarrow C, C \rightarrow DA\}$

[b] $R(A,B,C,D) \{B \rightarrow C, BC \rightarrow DA\}$

CK: [1] B

Highest NF: BCNF

Reason: The LHS of all non-trivial FS are superkeys.

Canonical Cover (optional): $\{B \rightarrow ACD\}$

[c] $R(A,B,C,D) \{B \rightarrow C, A \rightarrow C, AC \rightarrow D\}$

CK: [1] AB

Highest NF: 1NF

Reason: $B \rightarrow C$, $A \rightarrow C$ and $A \rightarrow D$ all violate 2NF as the LHS are a proper subset of a CK (AB) and the RHS are non-prime

Canonical Cover (optional): $\{B \rightarrow C, A \rightarrow CD\}$

[d] $R(A,B,C,D) \{B \rightarrow C, C \rightarrow AB, AB \rightarrow D\}$

CK: [1] B, [2] C

Highest NF: BCNF

Reason: The LHS of all non-trivial FS are superkeys.

Canonical Cover (optional): $\{B \rightarrow C, C \rightarrow ABD\}$

[4] {RS->PQ, QR->S, S->P, U->ST, STU->R, Q->U}

(a)

P+=P

Q+= PQRSTU

R+ = R

S+ = SP

T+ = T

U+ = PQRSTU

(b) CK: [1] Q, [2] RS and [3] U.

(c) Prime: Q,R,S,U; non-prime: P,T

(d) Canonical Cover: there are many, for examples, {RS-> Q, Q->RSTU, S-> P, U->Q}

(e) Highest NF: 1NF. S->P violates the 2NF.

(f) The decomposition into R1(Q,R,S,T,U) {RS-> Q, Q->RSTU, U->Q} and R2(P,S) {S-> P} are lossless and FD preserving. R1 and R2 are in BCNF. Further decomposition of R1(Q,R,S,T,U) is also acceptable but not desirable.

[5] For Employee(Eid, EFName, ELName, EPhone, DeptId, DeptName, PositionId, RateScale)

[a] Functional Dependencies (as in a canonical cover):

1. Eid -> Eid, EFName, ELName, PositionId
2. DeptId -> DeptName
3. DeptName -> DeptId (likely)
4. PositionId -> RateScale

[b] The CKs are

1. Eid, DeptId, EPhone
2. Eid, DeptName, EPhone

[c] Thus, Highest normal form is 1NF as Eid -> Eid, EFName, ELName, PositionId violate 2NF, for example.

[d] Decomposition:

Employee(Eid, EFName, ELName, PositionId)

1. FD: [1] Eid -> Eid, EFName, ELName, PositionId
2. FK: [1] PositionId references Position(PositionId)
3. Highest NF: BCNF

EmployeePhone(Eid, EPhone)

1. FD: {}
2. FK: [1] Eid references Employee(Eid)
3. Highest Normal Form: BCNF

Department(DeptId, DeptName)

1. FD: [1] DeptId -> DeptName, [2] DeptName -> DeptId
2. FK: {}
3. Highest NF: BCNF

WorkFor(DeptId, Eid)

1. FD: {}
2. FK: [1] DeptId references Department(DeptId), [2] Eid references Employee(Eid)
3. Highest NF: BCNF

Position(PositionId, PayScale):

1. FD: [1] PositionId -> PayScale
2. FK: {}
3. Highest NF: BCNF

[6] Minimum: 4, when there is only one candidate key: ABC.

Maximum: 28 when there are three candidate keys: [1] A, [2] B and [3] C.