

**Database Systems**  
**Fall 2025**  
**Suggested Solution for Homework #7**

[1] See h7q1\_sol.pdf.

[2]  $F = \{A \rightarrow B, C \rightarrow D, AB \rightarrow E, CE \rightarrow F\} \mid - AC \rightarrow F$

Proof.

- [1]  $A \rightarrow B$  (given)
- [2]  $AB \rightarrow E$  (given)
- [3]  $A \rightarrow E$  (pseudo-transitivity rule on [1] and [2] and simplification)
- [4]  $CE \rightarrow F$  (given)
- [5]  $AC \rightarrow E$  (pseudo-transitivity rule on [3] and [4])

[3]

- [a]  $R(A,B,C,D) \{D \rightarrow C, C \rightarrow B\}$
- [b]  $R(A,B,C,D) \{AB \rightarrow C, C \rightarrow D\}$
- [c]  $R(A,B,C,D) \{A \rightarrow B, B \rightarrow ACD\}$
- [d]  $R(A,B,C,D) \{AB \rightarrow C, AD \rightarrow C\}$
- [e]  $R(A,B,C,D) \{A \rightarrow B, B \rightarrow A, AC \rightarrow D\}$

[a]  $R(A,B,C,D) \{D \rightarrow C, C \rightarrow B\}$   
CK: [1] AD  
Highest NF: 1NF  
Reason:  $D \rightarrow C$  violates 2NF since D is a proper subset of a CK (AD), and C is non-prime.

[b]  $R(A,B,C,D) \{AB \rightarrow C, C \rightarrow D\}$   
CK: [1] AB  
Highest NF: 2NF  
Reason:  $C \rightarrow D$  violates 3NF as C is not a superkey, and D is non-prime.

[c]  $R(A,B,C,D) \{A \rightarrow B, B \rightarrow ACD\}$   
CK: [1] A, [2] B  
Highest NF: BCNF  
Reason: A and B in the LHS of FDs are superkeys.

[d]  $R(A,B,C,D) \{AB \rightarrow C, AD \rightarrow C\}$   
CK: [1] ABD  
Highest NF: 1NF  
Reason:  $AB \rightarrow C$  violates 2NF since AB is a proper subset of ABC, a CK, and C is non-prime.

[e]  $R(A,B,C,D)$   $\{A \rightarrow B, B \rightarrow A, AC \rightarrow D\}$

CK: [1] AC, [2] BC

Highest NF: 3NF

Reason:  $A \rightarrow B$  and  $B \rightarrow A$  both violate BCNF as they are not superkeys.

[4] Consider  $F = \{A \rightarrow B, BC \rightarrow D, AB \rightarrow E, E \rightarrow AB\}$

(a)  $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow E, E \rightarrow A$

(b) The candidate keys are [1] AC, [2] EC

(c) Prime: A, C, E, non-prime: B, D

(d) For example,  $\{A \rightarrow BE, BC \rightarrow D, E \rightarrow A\}$  or  $\{A \rightarrow E, BC \rightarrow D, E \rightarrow AB\}$

(e) 1NF since  $A \rightarrow B$  violates 2NF: B is non-prime and A is a proper subset of a CK.

(f) Yes, the decomposition:

1.  $R_1(A,B,E)$   $\{A \rightarrow BE, E \rightarrow A\}$

2.  $R_2(B,C,D)$   $\{BC \rightarrow D\}$

3.  $R_3(A,C)$   $\{\}$

[5] [5] For Tutor(TutorId, TLName, TFName, StudentId, SLName, SFName, SubjectId, SubjectName, StartDate, Level).

[a] Functional Dependencies:

TutorId  $\rightarrow$  TLName, TFName

StudentId  $\rightarrow$  SLName, SFName

SubjectId  $\rightarrow$  SubjectName

SubjectName  $\rightarrow$  SubjectId

TutorId, StudentId, SubjectId, Level  $\rightarrow$  StartDate

[b] The CKs are

1. TutorId, StudentId, SubjectId, Level

2. TutorId, StudentId, SubjectName, Level

[c] Thus, the highest normal form is 1NF as TutorId  $\rightarrow$  TLName, TFName violates 2NF, for example.

[d] Decomposition:

Tutor(TutorId, TLName, TFName) {TutorId  $\rightarrow$  TLName, TFName}; BCNF

Student(StudentId, SLName, SFName) {StudentId  $\rightarrow$  SLName, SFName}; BCNF

Subject(SubjectId, SubjectName) {SubjectId  $\rightarrow$  SubjectName, SubjectName  $\rightarrow$  SubjectId}; BCNF

TutorAssignement(TutorId, StudentId, SubjectId, Level, StartDate) {TutorId, StudentId, SubjectId, Level - > StartDate}; BCNF

[6] It is known that for R(A,B,C,D,E):

1. R has exactly two candidate keys
2. A is a candidate key.
3. D and E are non-prime attributes.

There are three scenarios for the second candidate keys:

1.  $B \Rightarrow 24$  SK {A, AB, AC, AD, AE, ABC, ABD, ABE, ACD, ACE, ADE, ABCD, ABCE, ABDE, ACDE, ABCDE, B, BC, BD, BE, BCD, BCE, BDE, BCDE}
2.  $C \Rightarrow 24$  SK {You figure out yourself.}
3.  $BC \Rightarrow 20$  SK {A, AB, AC, AD, AE, ABC, ABD, ABE, ACD, ACE, ADE, ABCD, ABCE, ABDE, ACDE, ABCDE, BC, BCD, BCE, BCDE}

Thus, the number SK: {20,24}