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**Theory of Functional Dependency**

by K. Yue

**1. Armstrong's axioms**

* Armstrong's axioms is an inference system.
* Reasoning from the *definition*is difficult.

A relation *schema* R is said to *satisfy* the *functional dependency* X -> Y if for *any* relation *instance* r that uses R, if there exists two tuples s and t ∈ r such that s[X] = t[X], then s[Y] = t[Y].

∃: there exists

=> implies

->: FD, determines.

1. (∃s, t ∈ r) (s[X] = t[X]) => s[Y] = t[Y]
2. i.e.  same value in X implies same value in Y.

* Thus, people invented axioms as an equivalent way for*inference*.
* A set of *axioms* for inference with FD: <http://en.wikipedia.org/wiki/Armstrong%27s_axioms>.
* Axioms: 'self-evidence' or 'assumed' so that they can be used as the basis of inference.
* Three basic axioms:
  1. *Reflexivity:* If X and Y are sets of attributes and Y is a subset of X, then X -> Y. (e.g. AB -> A; {FName, LName} -. FName)
  2. *Augmentation*: If X -> Y then X Z -> Y Z (LHS and RHS are both augmented by Z) (e.g. IF AB->C then ABD->CD) (IF StuID -> LName, then {StuId, grade} -> {LName, Grade}
  3. *Transitivity***:**If X -> Y and Y -> Z then X -> Z (e.g. EMpId -> DeptId, DeptId -> DepName => EmpId -> DepName}
* Three additional rules that can be derived from the basic axioms.
  1. *Pseudo-transitivity* Rule: If X-> Y, YZ -> A then XZ -> A
  2. *Decomposition* Rule: If X -> Y Z, then X -> Y and X -> Z. (Note that the decomposition applies to the RHS of the FD)
  3. *Union* Rule:  If X -> Y and X -> Z then X -> Y Z. (Note that the decomposition applies to the RHS of the FDs)
* Armstrong's axioms are *sound* and *complete*.
  1. Soundness: Inference using the axioms will create only correct FD.
  2. Completeness: The axioms can be used to derive *all* correct FD.
* Computing students need to know how to infer using a formal mathematical method.

***Example***

Let X be CITY STREET, Y be STREET, then Y is a subset of X, and X -> Y or CITY STREET -> STREET (Reflexivity axiom).

* If two tuples have the same values of CITY and STREET, then they surely have the same value of STREET.
* This is so trivial that we call a functional dependency likes CITY, STREET -> STREET a *trivial functional dependency*. They do not actually specify any problem requirement but are mathematical true.

***Example:***

For R(A,B), we have the following trivial FD for the attributes A and B. No matter what A and B are supposed to mean, they are always mathematically true. (Φ is the empty set.)

AB -> AB, AB->A, AB->B, AB-> Φ  
A -> A, A-> Φ  
B -> B, B-> Φ

Remember AB-> AB means {A, B} -> {A, B}.

* Since trivial functional dependencies do not actually represent any problem requirements, we are only interested in *non-trivial* functional dependency. Non-trivial FD are FD in which its RHS is not a subset of its LHS.

If EmpId  ->  DeptId, and DeptId  ->  ManagerId  
then EmpId  ->  ManagerId.

Interpretation: If

1. every Employee works for only one department, and
2. every department has only one manager,

then every Employee has only one manager.

A non-trivial FD X->Y:

1. Y is not a subset of X.
2. It represents problem requirements.
3. It is not universally true. It may be false under a different set of problem requirements.

A mathematical proof using Armstrong's axiom is to continuously create/prove new FDs until the result is included. Reasons are usually given.

***Example***

Prove that the decomposition rule is true: X->YZ => X->Y and X->Z

Proof:

[1] X->YZ (given)  
[2] YZ -> Y (reflexivity axiom)   
[3] X -> Y (transitivity axiom on [1] and [2]).  
[4] YZ -> Z (reflexivity axiom)  
[5] X -> Z (transitivity axiom on [1] and [4]).

To prove Pseudo-transitivity Rule: If X-> Y, YZ -> A then XZ -> A

[1] X -> Y (given)  
[2] XZ -> YZ (augmentation axiom adding Z to [1])  
[3} YZ -> A (Given)  
[4] XZ -> A (transitivity axiom on [2] and [3]

**2. Keys and Superkeys Revisited**

* We can use the concept of FD to define keys and superkeys.
* For a relation scheme R, K is a candidate key (CK) if
  1. Uniqueness:  K -> R. (e.g. K: {stuId} in student)
  2. Minimality:  there is no *proper* subset of K that determines R. (There is no *extraneous* attribute.) e.g. {StuId}, {stuId, LName} … are SK
* Note that
  1. |A| = cardinality of A = the number of elements in the set A.
  2. A ⊆ B means A is a subset of B and it is possible that A = B.
  3. A ⊂B means A is a proper subset of B in which A <> B.
  4. If A ⊆ B, |A| <= |B|.
  5. If A ⊂ B, |A| < |B|.
* K is a superkey if K -> R.
* Superkeys (SK) do not need to satisfy the minimality requirement.
* Some properties:
  1. If K is a CK, any superset of K is a SK.
  2. If K is a CK, any proper subset of K is not a CK (not unique)
  3. If K is a CK, any proper superset of K is not a CK (not minimal).
* Note that the primary key of a table is just a selected candidate key used to structure the p*hysical storage*. PK has the same logical properties like other candidate keys (*alternate or secondary keys*) in the context of the normalization theory.
* A CK with only one attribute is known as a *simple key*.
* A CK with more than one attributes is known as a *composite key*.
* A *compound* key is a composite key in which every component attribute is a foreign key. E.g {stuId, classId} in enroll.

***Example***

In Employee(EmpId, DeptId, ManagerId) with

EmpId -> DeptId, and  
DeptId -> ManagerId.

By the transitivity axiom, EmpId -> ManagerId  
By the union rule, EmpId -> EmpId, DeptId, ManagerId  
By the augmentation axiom, EmpId, ManagerId -> DeptId, ManagerId  
                         
Hence, EmpId is a CK of Employee(EmpId, DeptId, ManagerId).

On the other hand,

1. DeptId is not a candidate key since we do not have DeptId -> EmpId.
2. {Empd, DeptId} is not a candidate since it is not minimal. It is a superkey only.

Furthermore, there are four superkeys:

1. EmpId
2. EmpId, DeptId
3. EmpId, ManagerId
4. EmpId, DeptId, ManagerId

**3. Finding ALL Candidate Keys**

**3.1 Closure of Attributes**

* Given a set of FD F, the *closure* of a*set of attributes* X, denoted as X+, is the set of all attributes functionally determined by X using Armstrong's axioms on F.

***Example***

Consider R(A,B,C,D) with

F = {B->A, A->C, AB->D, D->AC}

A+: A  
 : A C (A is in A+; A-> A, A->C => A-> C}

A+ = AC

F = {B->A, A->C, AB->D, D->AC}

B+: B  
 : B A  
 : B A C {B+ contains A; B-> A, A -> C => B -> C by transitivity axiom)  
 : B A C D (B+ contains AB; B-> AB, AB->D => B -> D by transitivity axiom)

B+ = ABCD = R (B is a CK)  
C+ = C  
D+ = ACD

Thus, B is a candidate key (CK).

No proper superset of B is a candidate key (since it will not be minimal).

Remaining non-empty subsets of ABCD to check for candidate keys:

AC+ = AC  
AD+ = ACD  
CD+ = ACD  
ACD+ = ACD

Thus, B is the only CK.

Prime/key attribute: B (appear in some CK)  
Non-prime/non-key attributes (does not appear in any CK): A, C, D

* The closure of attributes can be used for other purposes, such as checking validity of FD, computing closures of a set of functional dependencies, checking equivalence of two set of FDs, etc.

**3.2 Algorithm for finding X+ for a set of FDs F.**

[1] X+ <- X // Start with X in X+ because X -> X. (reflexivity axion)  
[2] while (  
         [A] there exists a FD P -> Q such that  
         [B] P is a subset of X+, ( X -> P) and (X -> Q)  
         [C] there are attributes K in Q not in X+) {  
   [3] X+ <- X+ U Q      // Add attributes in Q to X+ by using the union operator.  
}

**3.3 Finding ALL Candidate keys (NP-Complete Problem)**

* It is necessary to find *all* candidate keys to conduct normalization analysis.
* In general, if R has n attributes, there are 2n - 1 subsets of R which are potential candidate keys.

***Example:***

For R(A,B,C), need to check A, B, C, AB, AC, BC and ABC for candidate keys.

Thus, the problem of finding all candidate keys in R is O(en), where n is the number of attributes in the relation R.

**3.4 To find all candidate keys of R with a set of FD, F:**

1. Find the *canonical cover*, FC, first. This simplifies F. (See later)
2. Use *heuristics* to cut down the number of sets of attributes to check for candidate keys.
3. Classify attributes into three groups:
   1. L/NR (left only or not right): If an attribute X does not appear in the right hand side (RHS) of any f in F, *every* candidate key must include X.
   2. R (right only): If X appears only in the RHS of a fd in F but does not appear in the LHS of any f in F, then x is *not* a component of *any* candidate key.
   3. M (mixed; left and right): If X appears in LHS in some FD and in RHS in some other FD in F, then X *may* potentially be in some CK.
4. If X is found to be a CK, then any proper superset of X is not a CK, and needs not be checked.

***Example:***

Consider R(A,B,C,D) with

F = {B->A, A->C, AB->D, D->AC}

We have:

L/NR: B (in every CK)  
M: A, D (may be in some CK)  
R: C (not in any CK)

Checking: B and then BA, BD, BAD (if needed).

B+: BACD

Thus, there is only one CK: [1] B.

[4] (20%) Consider R(P, Q, R, S, T, U) with F = {P->Q, R->S, PQ->ST, U->RS, UP->S}

(a) What are P+, Q+, R+, S+, T+, U+?

with F = {P->Q, R->S, PQ->ST, U->RS, UP->S}

P+: P Q ST (closure of a set of attributes)  
…

(b) What are the candidate keys?

L/NR: P U (in every CK)  
M: Q R Q   
R: S T (not in any CK)

PU+: PU Q ST R

Check PU and PUQ, PUR, PUQR

CK: [1] PU

(c) Show all prime attributes and non-prime attributes.

Prime: P, U  
Non-prime: Q, R, S, T

(d) Provide a canonical cover of F.

with F = {P->Q, R->S, PQ->ST, U->RS, UP->S}

F1 = {P->Q, R->S, PQ->S, PQ ->T, U->R, R->S, UP->S}

Extraneous attribute:

Is P extraneous in PQ -> S? No  
Q+: Q (Q -x-> S)

Is Q extraneous in PQ -> S? Yes  
P+: P Q ST (P-> S)

F2 = {P->Q, R->S, P->S, PQ ->T, U->R, R->S, UP->S}

Is P extraneous in PQ ->T? No  
Q+: Q (Q -x-> T)

Is Q extraneous in PQ -> T? Yes  
P+: P Q ST (P-> T)

F3 = {P->Q, R->S, P->S, P ->T, U->R, R->S, UP->S}

Is P extraneous in UP->S? Yes  
U+: U R S (U -> S)

F4 (set of FD) = {P->Q, R->S, P->S, P ->T, U->R, R->S, U->S}

Check for redundant FD:

F4 (set of FD) = {P->Q, R->S, P->S, P ->T, U->R, U->S}

Is P->Q redundant,

FA = {R->S, P->S, P ->T, U->R, U->S} |- P->Q? No

In FA: P+: P S T (P -x-> Q); FA is not a cover of F4

Is U-> S redundant?

FB = {P->Q, R->S, P->S, P ->T, U->R } |- U ->S? Yes

In FB: U+: U R S

FB = {P->Q, R->S, P->S, P ->T, U->R}

Canonical Cover = {P->QST, U->R, R->S}

(e) What is the highest normal form (up to BCNF) of R? Why? (f) If R is not in BCNF, provide a lossless FD preserving decompositions of R into BCNF relations?

**4. FD Closure and Covers (If time permits)**

**4.1 Closure of a set of functional dependencies (FD)**

* The *closure* of a set of FD, F, is denoted by F+, and is the set of all FDs that are*logically implied* by F.

Consider F = {A->B, B->C}

F+ = {  
A->{}, A->A, A->B, A->C, A-> AB, A-> AC, A-> BC, A->ABC,  
B->{}, B->B, B->C, B->BC,  
C->{}, C->C,  
AB->{}, AB->A, AB->B, AB->C, AB->AB, AB->AC, AB->BC, AB->ABC,  
AC->{}, AC->A, AC->B, AC->C, AC->AB, AC->AB, AC->BC, AC->ABC,  
BC->{}, BC->B, BC->C, BC->BC,  
ABC->{}, ABC->A, ABC->B, ABC->C, ABC-> AB, ABC-> AC, ABC-> BC, ABC->ABC }

Note that

* Many FDs in F+ are trivial. Examples: A->{}, ABC->AC, etc.
* FD+ itself is not very interesting.

**4.2 Equivalence and cover**

* Two sets of FD, F and G are *equivalent*, if F+ = G+. They are *covers* of each other.
* Thus, covers can be used to support the concepts of equivalence.
* If F and G are covers of each other, they represent the same set of application requirements and assumptions.

**4.3 Canonical and Minimal Covers**

* **Definition.** In a set of FDs F, the attribute A in the FD P-> Q is *extraneous* if F - {P-> Q} U {P-A -> Q} is equivalent to F.
* Thus, the attribute A is not actually needed in P to determine Q. It is extraneous.

***Example***

Consider the F = {A->B, AB->C}.

B is extraneous since for G = {A->B, A->C}, and F+ = G+.

* **Definition**. A FD f in F is *redundant* if (F - f)+ = F+.

***Example***

In F = {A->B, AB->C, B->C},

AB->C is redundant since for

G = {A->B, B->C}, AB+ = ABC.

Alternatively, we may state that

G |- AB-> C.

***Example***

For F = {A->BC, B->C}

Using decomposition rule,

F' = {A->B, A->C, B->C} is a cover of F.

In F', A->C is redundant since {A->B, B->C} |- A->C

Thus F" = {A->B, B->C} is a cover of F' and F.

* **Definition.** A *canonical cover*, G, of F satisfies the following conditions:
  1. G is a cover of F; G+ = F+.
  2. There is no redundant FD in G.
  3. There is no extraneous attribute in G.
  4. The left hand side (LHS)of every FD in G is unique.
* **Definition.** A *minimal cover*, G, of F satisfies the following conditions:
  1. G is a cover of F; G+ = F+.
  2. There is no redundant FD in G.
  3. There is no extraneous attribute in G.
  4. The right hand side (RHS) of every FD in G contains only a single attribute

In F = {A->B, AB->C, B->C, A->D},

G1 = {A->B, B->C, A->D} is a minimal cover, not canonical

G2 = {A->BD, B->C} is a canonical cover, not minimal.

* The minimal covers and canonical covers are *simplified* equivalent versions of a set of FDs, representing the same set of data constraints.
* They are useful in understanding FD and for proper decompositions to remove unnecessary redundancy.

***Example:***

Consider F: {A->C, BCD->A, C->E, CD-> A, AB->C}

[1] Does F imply BD-> A (i.e. F |- BD -> A)?

No, Since in F, BD+ = BD

Thus, C is not extraneous in BCD -> A.

[2] F |- AE -> B ?

No, since AE+ = AE C

[3] Give a canonical cover for F.

{ A->C, CD->A, C->E }

[4] Show all candidate keys.

L/NR: B, D  
M: A, C  
R: E

CK: [1] ABD, [2] CBD

***Example (Tedious):***

Find a canonical cover for F = {BC->AE, AD->BCE, A->E, AE->D, BCD->F, AB->C}

***Solution:***

Basically, we iteratively remove all extraneous attributes and redundant function dependencies.

We use decomposition rule to ensure the RHS to contain only a single attribute so we can work on them one by one. F becomes:

(1) BC -> A  
(2) BC -> E  
(3) AD -> B  
(4) AD -> C  
(5) AD -> E  
(6) A -> E  
(7) AE -> D  
(8) BCD -> F  
(9) AB -> C

To investigate whether B or C is extraneous in BC -> A, we note that in F:

B+ = B  
C+ = C

This means B alone and C alone cannot determine A, and neither of them is extraneous.

On the other hand, in F:

A+ = ABCDEF

That means A alone can determine all other attributes. Any other attributes in the LHS with A in a FD are thus extraneous, we thus have the following by removing D in [2], [3] and [4], and B in [9].

(1) BC -> A  
(2) BC -> E  
(3) A -> B  
(4) A -> C  
(5) A -> E  
(6) A -> E  
(7) A -> D  
(8) BCD -> F  
(9) A -> C

Removing identical FD, we have F:

(1) BC -> A  
(2) BC -> E  
(3) A -> B  
(4) A -> C  
(5) A -> E  
(6) A -> D  
(7) BCD -> F

For (7), since B+ = B, C+ = C and D+ = D. However, BC+ = ABCDEF, and thus D is extraneous. Thus, we now have:

(1) BC -> A  
(2) BC -> E  
(3) A -> B  
(4) A -> C  
(5) A -> E  
(6) A -> D  
(7) BC -> F

To check for redundant FD, we consider whether we can deduce the FD when it is removed.

For (1) BC -> A, removing it result in F':

(1) BC -> E  
(2) A -> B  
(3) A -> C  
(4) A -> E  
(5) A -> D  
(6) BC -> F

In F': we have

BC+ = BCE, which does not include A. Thus, F' does not imply BC -> A and it is not redundant.

For (2) BC -> E, removing it and we have F':

(1) BC -> A  
(2) A -> B  
(3) A -> C  
(4) A -> E  
(5) A -> D  
(6) BC -> F

In F', we have BC+ = ABCDEF. Thus, F' |= BC -> E and BC -> E is redundant. Remove it and we have:

(1) BC -> A  
(2) A -> B  
(3) A -> C  
(4) A -> E  
(5) A -> D  
(6) BC -> F

Using this method, we can find that there are no more redundant FD.

Finally, we use the union rule to merge FD with the same LHS and get the canonical cover:

{BC -> AF, A-> BCDE}

Note that the canonical cover is not unique. Another canonical cover is:

{BC -> A, A-> BCDEF}

***Exercise:***

Consider F: {AB->CE, BC->D, D->BC, C->E, A->C, A->E}

Find:

* all candidate keys.
* a canonical cover of F.

***Exercise:***

Can there be more than one canonical covers for a set of FDs?

**Normal Forms and Theory of Normalization**

by K. Yue

**1. Normal Forms Using Functional Dependencies**

**1.1 First Normal Form**

* A relation is in 1NF if all attribute values are *atomic*: no repeating group, no composite attributes, no internal structures.
* Semantically, an attribute is atomic if it cannot be broken down to smaller pieces with individual meanings.
* Theoretically, a relation may only have atomic attributes.  Thus, all pure 'relations' satisfy 1NF.
* In practice, DBMS may allow data types with composite values and internal structures, e.g. set, JSON, spatial, XML, etc.

***Example***

Consider the following table with 3 records. It is not in 1 NF.

* The value "10000, 12000, 13000" of the field EmpIds can be broken into three components, "10000", "12000", and "13000" with individual meanings.
* The same is true for the field Names.
* Note the plural forms of EmpIds and Names.

|  |  |  |  |
| --- | --- | --- | --- |
| **DeptId** | **ManagerId** | **EmpIds** | **Names** |
| D123 | 110 | 10000, 12000, 13000 | Lady Gaga, Eminem, Lebron James |
| D225 | 440 | 21000, 22000 | Rajiv Gandhi, Bill Clinton |
| D337 | 300 | 31000 | John Smithson |

An alternate design of the relation in 1NF is shown below. The following instance has six rows.

|  |  |  |  |
| --- | --- | --- | --- |
| **DeptId** | **ManagerId** | **EmpId** | **Name** |
| D123 | 110 | 10000 | Lady Gaga |
| D123 | 110 | 12000 | Eminem |
| D123 | 110 | 13000 | Lebron James |
| D225 | 440 | 21000 | Rajiv Gandhi |
| D225 | 440 | 22000 | Bill Clinton |
| D337 | 300 | 31000 | John Smithson |

* Why atomic?
  + Relational theory and operations treat attributes as atomic.
  + No need to worry about the correctness and consistency of internal structures.
* Relations not satisfying 1NF have undesirable redundancy and anomalies.

***Example***

Consider the tuple (EmpId: 12345, OSSkills: {Windows, Linux, Solaris}).

* It will be difficult to identify all Employees with Linux skills.
* It will be difficult to join other tables using OSSkills.
* Data entry problems and issues, e.g. Linux linux, linx, etc., may further degrade data quality and introduce inconsistency.
* On the other hand, relations may be in Non-First Normal Form (NFNF of NF2), mainly for *performance* considerations. E.g., NoSQL DB.

**1.2 Second Normal Form**

* A relation R is in 2NF if
  1. R is in 1NF, and
  2. all *non-prime* attributes are *fully* dependent on the *candidate* keys.
* Review: A prime attribute (also called key attribute) appears in one or more candidate keys. Otherwise, it is a non-prime (non-key) attribute. Note that a relation may have many candidate keys.
* There is *no partial dependency* in 2NF.

***Example***

The following relation is not in 2NF.  (Assume that the number of credits of a given course does not change). Note the redundancy and anomalies.

Enroll(Course, Credit, Student, Grade)

|  |  |  |  |
| --- | --- | --- | --- |
| ***CourseId*** | ***Credit*** | **StudentId** | **Grade** |
| *C1* | *3* | S1 | A |
| *C1* | *3* | S2 | B |
| *C1* | *3* | S3 | B |
| C2 | 2 | S1 | A |
| C2 | 2 | S4 | D |

L/NR: CourseId, StudentId; R: Credit, Grade; M: N/A

We assume the following FD. (not in 2NF)

1. CourseId (a part of a CK; a proper subset of a CK) -> Credit (non-prime): violates 2NF; locate the unnecessary redundant.
2. CourseId, StudentId (full CK) -> Grade (non-prime): Ok with 2NF
3. {CourseId, StudentId} is the only candidate key.
4. Prime attributes: CourseId, StudentId
5. Non-prime attribute: Credit, Grade.
6. FD (1), CourseId -> Credit, is a partial dependency that violates 2NF.
   * CourseId (a proper subset of a CK) -> Credit (a non-prime attribute)

To convert to relations in 2NF, decompose Enroll into

1. Enroll(CourseId, StudentId, Grade) {CourseId, StudentId -> Grade}
2. Class(CourseId, CreditId) {CourseId -> Credit}: CK: CourseId

Both tables are in 2NF.

1. {CourseId -> Credit} violates 2NF in Enroll(CourseId, Credit, StudentId, Grade)
   * CourseId is a proper subset of a CK (CourseId, StudentId) in Enroll
2. {CourseId -> Credit} does not violate 2NF in Class(CourseId, Credit)
   * CourseId is a CK in Class. It is not a proper subset of a CK.

***Example from Ricardo:***

NewClass(courseNo, stuId, stuLastName, facId, schedule, room, grade). We have:

courseNo, stuId –(full CK) > grade (non-prime); ok with 2NF  
stuId (a part of CK) -> stuLastName (non-prime): violates 2NF  
courseNo (non-prime)-> facId, schedule, room (non-prime) ): violates 2NF

Highest NF: 1NF  
  
StuId -> stuLastName and courseNo -> facId, schedule, room violate 2NF

To convert to 2NF, decomposition:

1. Course(courseNo, facId, schedule, room) { courseNo -> facId, schedule, room } The FD is no longer violating 2NF in the new table Course since courseNo is a CK in Course.
2. Student(stuId, stuLastName) { StuId -> stuLastName } The FD is no longer violating 2NF in the new Student table since StuId is a CK in Student
3. Enroll(courseNo, stuId, grade) { courseNo, stuId -> grade }

**1.3 Third Normal Form**

* (New definition) A relation R is said to be in the third normal form if for every *non-trivial* functional dependency X -> A,
  1. X is a superkey, *or*
  2. A is a *prime* (key) attribute.
* (Old definition: included for historical reason. Do not use it for normalization analysis.) A relation R is in 3NF if
  1. R is in 2NF, and
  2. There is no *transitive* dependency of *non-prime* attributes on the candidate keys.
* Review: A superkey S of a relation R satisfies S -> R (uniqueness). It does not need to satisfy the minimal property.
* 3NF can identify unnecessary redundancy that 2NF cannot identify.
* 3NF still cannot eliminate all redundancies due to functional dependencies.

***Example***

* The following relationis in 2NF, but is not in 3NF.

|  |  |  |  |
| --- | --- | --- | --- |
| **DeptId** | **ManagerId** | **EmpId** | **Name** |
| *D123* | *110* | 10000 | Lady Gaga |
| *D123* | *110* | 12000 | Eminem |
| *D123* | *110* | 13000 | Lebron James |
| D225 | 205 | 21000 | Rajiv Gandhi |
| D225 | 205 | 22000 | Bill Clinton |
| D337 | 333 | 31000 | John Smithson |

* If we assume the following canonical set of FDs:
  1. EmpId (CK, also a SK) -> Name, DeptId (non-prime): OK with 3NF
  2. DeptId (not a SK) -> ManagerId (non-prime): violates 3NF
* Then:
  1. There is only one candidate key: EmpId
  2. Prime attribute: EmpId
  3. Non-prime attributes: Name, DeptId, ManagerId.
  4. The relation is in 2NF.(If all CK are simple, then in 2NF)
* The relation is not in 3NF because:
  1. EmpId is the only candidate key.
  2. EmpId is prime.
  3. DeptId and ManagerId are non-prime.
  4. DeptId -> ManagerId violates 3NF because:
     1. DeptId is not a SK.
     2. ManagerId is non-prime.
* To resolve, decompose the relation into:
  1. Department(DeptId, MangaerId) { DeptId (SK) -> ManagerId (non-prime)}: PK: DeptId
  2. Employee(EmpId, Name, DeptId) { EmpId -> Name, DeptId }

***Example***

Consider the relation R(CITY, STREET, ZIP) with the FDs:

1. CITY STREET -> ZIP, and
2. ZIP -> CITY.

There are two candidate keys:

1. CITY STREET, and
2. ZIP STREET

Hence, all attributes are prime attributes, and the relation is in both 2NF and 3NF.

Note that a relation such as Employee(EmePId, Name, Street, City, Zip, State) is not in 3NF.

This is a classical example you can find in many database textbooks. Note that the two FDs may not actually be correct in the United States.

* 3NF does not eliminate all redundancies due to functional dependencies.

**1.4 BCNF (Boyce-Codd Normal Form)**

* A relation R is said to be in **BCNF** if for *every* *non-trivial* functional dependency X -> Y in R, X is a *superkey*.

***Example***

Consider the relation

S(SId, PId, SName, Quantity) with the following assumptions:

1. SId is unique for every supplier.
2. SName is unique for every supplier.
3. Quantity is the *accumulated* quantities of a part supplied by a supplier. Given a supplier and a part, the Quantity is unique.
4. A supplier can supply more than one part.
5. A part can be supplied by more than one supplier.

We have the following non-trivial FD:

1. SId -> SName
2. SName -> SId
3. SId PId -> Quantity
4. SName PId -> Quantity

Note that SId and SName are *equivalent*.

The candidate keys are:

1. SId PId
2. SName PId

Prime attributes: SId, PId, SName

Non-prime attribute: Quantity.

The candidate keys are:

1. SId PId
2. SName PId

Prime attributes: SId, PId, SName

Non-prime attribute: Quantity.

The relation is in 3NF. Note:

1. SId -> SName does not violate 3NF as SName is prime. Violates BCNF
2. SName -> SId does not violate 3NF as SId is prime. Violates BCNF
3. SId PId -> Quantity does not violate 3NF as {SId, PId} is a CK and also a SK.
4. SName PId -> Quantity does not violate 3NF as {SName, PId} is a CK and also a SK.
5. In 3NF; not in BCNF

However, there are unnecessary redundancy.

|  |  |  |  |
| --- | --- | --- | --- |
| ***Sid*** | ***SName*** | **PId** | **Quantity** |
| *S1* | *ABC* | P1 | 10 |
| *S1* | *ABC* | P2 | 20 |
| *S1* | *ABC* | P3 | 21 |
| S2 | DEF | P1 | 40 |
| S2 | DEF | P4 | 13 |
| S3 | XYK | P3 | 18 |

Thus, 3NF does not detect all design problems using FD.

However, S is not in BCNF because, for example, the functional dependency

SId -> SName is

1. non-trivial, and
2. SId is not a superkey.

To deal with it, we can decompose S(SId, PId, SName, Quantity) into

(1) Supplier(SId, SName) with

SId -> SName  
SName -> SId

with two candidate keys:

1. SId
2. SName

(2) Supply(SId, PId, Quantity)  with

SId, PId -> Quantity.

Both are in BCNF.

***Example:***

Consider the relation R(A, B, C, D) with

A -> B,  B -> C, C -> A and C -> D.

There are three candidate keys:

1. A
2. B
3. C

Since every left hand side of any non-trivial functional dependency is a superkey, R is in BCNF.

**1.5 Checking Highest Normal Form by Violations**

To find the highest normal form for a relation R, check every non-trivial FD X->Y of R for violation.

* Note that in the table below, A is a single attribute. Use the decomposition rule if necessary. For example, for AB->CD, check AB->C and AB->D.

Negation of the definitions of the NF:

|  |  |
| --- | --- |
| **Normal Form's*Violation*** | **Non-trivial FD X -> A** |
| 2NF | (1) X is a proper subset of a candidate key of R, and (2) A is a non-prime attribute. |
| 3NF | (1) X is not a superkey of R, and (2) A is a non-prime attribute. |
| BCNF | X is not a superkey. |

* If there is no violation of a normal form, then R is in that normal form.
* If there is one violation of a normal form, then R is not in that normal form.

***Example:***

Consider R(A,B,C,D) {A->B, B->AC, C->D}

Using decomposition rule, we have {A->B, B->A, B->C, C->D}

We find two CK: [1] A, [2] B  
Prime attributes: A, B  
Non-prime attributes: C, D

Checking for violation:

|  |  |  |  |
| --- | --- | --- | --- |
| **FD** | **Ok with 2NF** | **Ok with 3NF** | **OK with BCNF** |
| A (CK, SK) ->B (prime) | Yes | Yes | Yes |
| B (CK, SK) ->A (prime) | Yes | Yes | Yes |
| B (CK, SK) ->C (non-prime) | Yes | Yes | Yes |
| C (not a SK, not a proper subset of a CK) ->D (non-prime) | Yes | No | No |

Thus, the highest NF is 2NF

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[3] (20%) List the candidate keys and the highest normal forms for the following relations.

[a] R(A,B,C,D) {C->D, AC->B}

[b] R(A,B,C,D) {C-> AD, D->AB, A->B, B->A}

[c] R(A,B,C,D) {C->BD, BD->AC}

[d] R(A,B,C,D) {C->B, B->C, BD->A, CD->A}

Canonical cover: {C->B, B->C, BD->A }

CD -> A is a redundant FD

{C->B, B->C, BD->A } |- CD ->A? Yes. CD+: CD B A  
  
L/NR: D  
M: B, C  
R: A  
Check for CK: D, DB, DC and DBC  
D+: D  
BD+ = BD A C  
CD+: CD B A  
CK: [1] BD, [2] CD   
Prime: B, C, D  
Non-prime: A

|  |  |  |  |
| --- | --- | --- | --- |
| **FD** | **Ok with 2NF** | **Ok with 3NF** | **OK with BCNF** |
| C (not a SK) ->B (prime) | Yes | Yes | No |
| B (not a SK) ->C (prime) | Yes | Yes | No |
| BD (a CK, SK) -> A (non-prime) | Yes | Yes | Yes |

Highest NF: 3NF  
Decomposition:

[1] R1(B,C) {B->C, C->B}: BCNF  
[2] R2(B,D, A) {BD -> A}: BCNF

**1.6 Motivation of BCNF**

* The purpose of BCNF is to eliminate any unnecessary redundancy that FD can create in a relation.
  + In a BCNF relation, no value can be predicted from any sets of non-unique attributes, using *only* FD.
  + This is because in a BCNF relation, using FD only,
    - any attribute value can only be determined by a superkey,
    - but the superkey is unique.
  + However, there are other type of dependencies.
  + Therefore, there are higher normal forms.

***Example***

Consider the relation R(CITY, ZIP, STREET) again  
         
Using the code for the postal office, we have

CITY STREET -> ZIP, and ZIP -> CITY.

Hence, there are two candidate keys:

1. CITY STREET, and
2. ZIP STREET

Therefore, R is not in BCNF since in ZIP -> CITY, ZIP is not a superkey.

However, if we decompose R into two relations, each with two attributes, then the FD

CITY STREET -> ZIP is *lost* (i.e. cannot be assured within a *single* relation)

Therefore, we better leave the relation alone.

* Sometimes it is not possible for a relation to be in BCNF => need to settle in a less strict normal form (3NF).

**1.7 Normalization Theory Using Functional Dependencies**

* To use the theory on FD:
  1. For a relation of a set of attributes, we analyze the assumptions of the applications.
  2. From the assumptions, we obtain a set of FDs.
  3. Find a canonical cover of the set of FDs.
  4. Find all candidate keys, prime and non-prime attributes.
  5. If the relation is not in BCNF, we perform *decomposition*.
  6. If BCNF cannot be satisfied, we aim for 3NF.

***Example***

Consider the following relation:

Supply(SupplierId, SupplierName, ProductId, ProductDesc, Quantity, ArrivalTime)

The relation stores the quantities and arrival times of shipments of products (identified by ProductId) from suppliers (Identified by SupplierId). A supplier may not have a unique name. Furthermore, the product description, ProductDesc, may be the same for two products. A supplier may supply the same product many times, each with a different ArrivalTime.

The functional dependencies (FD) of the relation:

SupplierId -> SupplierName  
ProductId -> ProductDesc  
SuplierId, ProductId, ArrivalTime -> Quantity

CK:  {SupplierId, ProductId, ArrivalTime}

Non-prime attributes: SupplierName, ProductDesc, Quantity

Highest Normal Form: 1NF

SupplierId -> SupplierName violates 2NF since

1. SupplierId is a part (proper subset) of a candidate key, i.e., {SupplierId} ⊂ {SupplierId, ProductId, ArrivalTime}, and
2. Quantity is non-prime.

Not that A ⊂ B means that A is a proper subset of B.

**2. Decomposition**

* Decomposition is a major tool for constructing relations satisfying high enough normal forms.
* Decomposition should be disciplined:
  1. More relations may be less efficient in storage.
  2. More relations may be less efficient in executing queries.
* More importantly, some decompositions are harmful:
  1. *Lossy* decompositions.
  2. Decompositions that do *not preserve dependencies.*
* Hence, it is important to have *lossless dependency-preserving* decomposition (*good* decomposition).

**2.1 Lossy Decomposition**

***Example:***

Consider the relation Emp(EmpId, DeptId, ManagerId) with

EmpId ->  DeptId  
DeptId ->  ManagerId

Note that we do not have ManagerId -> DeptId in this example, since this organization allows a manager to manage more than one Departments. Note that ManagerId 90000 manages two Departments.

|  |  |  |
| --- | --- | --- |
| **EmpId** | **DeptId** | **ManagerId** |
| E1 | ACCT | M3 |
| E2 | HR | M3 |
| E3 | *ENG* | *M6* |
| E4 | *ENG* | *M6* |

The relation is not in 3NF because of the FD

DeptId (not a SK) -> ManagerId (non-prime)

Suppose we decompose the relation into

Emp1(EmpId, *ManagerId*)  
Dept(DeptId, *ManagerId*)  
  
The *common attribute* for the component relations is ManagerId. The relations are obtained by projections from Emp:

Emp1:

|  |  |
| --- | --- |
| **EmpId** | **ManagerId** |
| E1 | M3 |
| E2 | M3 |
| E3 | M6 |
| E4 | M6 |

Dept:

|  |  |
| --- | --- |
| **DeptId** | **ManagerId** |
| ACCT | M3 |
| HR | M3 |
| ENG | M6 |

If we do not *lose* any information by the decomposition, we should get the original relation using the natural join.

However,  Emp1 |x| Dept is

|  |  |  |
| --- | --- | --- |
| **EmpId** | **DeptId** | **ManagerId** |
| E1 | ACCT | M3 |
| *E1* | *HR* | *M3* |
| *E2* | *ACCT* | *M3* |
| E2 | HR | M3 |
| E3 | ENG | M6 |
| E4 | ENG | M6 |

This is not the same as the original relation Emp. Spurious rows are incorrectly included in the result.

Hence, the decomposition of Emp(EmpId, DeptId, ManagerId) into  
   
Emp1(EmpId, ManagerId) and  
Dept(DeptId, ManagerId)

is *lossy*.  It is not a good decomposition.

**2.2 Lossless Decomposition**

Example:

Consider now the following decomposition of Emp(EmpId, DeptId, ManagerId):

Emp2(*EmpId*, DeptId)  and  
Emp3(*EmpId*, ManagerId)

The common attribute is EmpId. We have Emp2 and Emp3:

Emp2:

|  |  |
| --- | --- |
| **EmpId** | **DeptId** |
| E1 | ACCT |
| E2 | HR |
| E3 | ENG |
| E4 | ENG |

Emp3:

|  |  |
| --- | --- |
| **EmpId** | **ManagerId** |
| E1 | M3 |
| E2 | M3 |
| E3 | M6 |
| E4 | M6 |

Hence, Emp2 |x| Emp3: (natural join)

|  |  |  |
| --- | --- | --- |
| **EmpId** | **DeptId** | **ManagerId** |
| E1 | ACCT | M3 |
| E2 | HR | M3 |
| E3 | ENG | M6 |
| E4 | ENG | M6 |

This is the same as the original relation Emp.  Therefore, the decomposition does not lose any information.  It is a *lossless*decomposition.

**Definition.** A decomposition is lossless if the natural joins of the component relations result in the original relation. Otherwise, it is lossy.

**2.3 Theory of Lossless Decomposition**

***Example:***

Why is the decomposition of Emp(EmpId, Dept, ManagerId) into

(1) Emp1(EmpId, ManagerId) and Dept(DeptId, ManagerId) *lossy*, and

(2) Emp2(EmpId, DeptId) and Emp3(EmpId, ManagerId) *lossless*?

**Theorem**: Suppose R(X, Y, Z) is decomposed into R1(X, Y) and R2(X, Z).  X is the set of common attributes in R1 and R2.  The decomposition is lossless if and only if

(a) X -> Y (X is a CK of R1), *or*  
(b) X -> Z (X is a CK of R2).

***Example:***

In case (1), X is ManagerId, Y is EmpId, Z is DeptId.

Neither condition (a) nor (b) is satisfied.  Hence, (1) is lossy.

In case (2), X is EmpId, Y is DeptId, Z is ManagerId.

Both conditions (a) and (b) are satisfied.  Hence, (2) is lossless.

* For decompositions into more than two relations, use the chase matrix algorithm, which is not covered in this course.

**2.4 Dependency-Preserving Decomposition**

***Example:***

For the relation Emp(EmpId, DeptId, ManagerId) with

EmpId ->  DeptId  
DeptId ->  ManagerId,

The decomposition of Emp into

Emp2(EmpId, DeptId)  and  
Emp3(EmpId, ManagerId)

is lossless but it does not *preserve dependencies*:

the FD  DeptId -> ManagerId

cannot be assured within a single relation after the decomposition. No relation contains both attributes.

For example, if we add the information Emp E6 work in the ACCT Department under manager M9 carelessly, we may have the following table.

* Since ACCT has M3 as manager already, it cannot also have M9 as its manager.

Emp2:

|  |  |
| --- | --- |
| **EmpId** | **DeptId** |
| E1 | *ACCT* |
| E2 | HR |
| E3 | ENG |
| E4 | ENG |
| *E6* | *ACCT* |

Emp3:

|  |  |
| --- | --- |
| **EmpId** | **ManagerId** |
| E1 | *M3* |
| E2 | M3 |
| E3 | M6 |
| E4 | M6 |
| *E6* | *M9* |

As a result, the FD  DeptId ->  ManagerId is violated.

* Department with DeptId ACCT has two ManagerId
  1. M3 (via EmpId E1)
  2. M9 (via EmpId E6)

Thus, for the relation Emp(EmpId, DeptId, ManagerId) with

EmpId ->  DeptId  
DeptId ->  ManagerId,

the best decomposition is

Emp1(EmpId, *DeptId*) (EmpId -> DeptId) and  
Dept(*DeptId*, ManagerId) (DeptId ->  ManagerId)

It is easy to show that, the decomposition is lossless, preserves dependencies, and that Emp1 and Dept are both in BCNF.

***2.4.2 Decomposition Algorithms***

1. It is possible to decompose a relation such that
   1. all member relations are in 3NF,
   2. the decomposition is lossless, and
   3. all FDs are preserved.
2. It is also possible to decompose a relation such that
   1. all member relations are in BCNF, and
   2. the decomposition is lossless, *but*
   3. not all FDs may be preserved.

**2.5 Algorithm for decomposition into 3NF relations**

* There are many algorithms for decomposition.
* We will not cover the details of the algorithms, but they are illustrated by the example below.
* In particular, the following examples show the steps of an lossless, FD preserving algorithm that guarantees 3NF.

***Example:***

Consider R(A,B,C,D,E) with F = {A->BC, CD -> E, BA -> C, D->B}.

Step 1. Find a *canonical cover* G for F.

The FD BA->C is redundant.

G = {A->BC, CD -> E, D->B}.

We may perform normalization analysis to see whether decomposition is necessary.

L/NR: A, D  
M: C  
R: B, E

We have: AD+ = AD BC E  
  
Thus, CK: [1] AD  
prime: A, D  
non-prime: B, C, E

Normalization analysis:

|  |  |  |  |
| --- | --- | --- | --- |
| **Non-trivial FD** | **2NF** | **3NF** | **BCNF** |
| A -> B: [1] A ⊂ AD, [2] A is not a SK, [3] B is non-prime | violate | violate | violate |
| A -> C: [1] A ⊂ AD, [2] A is not a SK, [3] C is non-prime | violate | violate | violate |
| CD -> E: [1] CD ⊄ AD, [2] C is not a SK, [3] E is non-prime | ok | violate | violate |
| D -> B: [1] D ⊂ AD, [2] D is not a SK, [3] B is non-prime | violate | violate | violate |

Thus, the highest normal form of R is 1NF. Decomposition is necessary.

Step 2. For every FD X->Y in G, create a relation with the schema XY and add it to the result D. This step preserves FD and resolves NF violations.

The FD BA->C is redundant.

G = {A->BC, CD -> E, D->B}.

Relations created:

R1(A,B,C) with A->BC  
R2(C,D,E) with CD->E  
R3(B,D) with D->B

It can be seen very easily that R1, R2 and R3 are all in 3NF and BCNF. Furthermore, all FDs are preserved.

Step 3. If no relation in D contains a candidate key of R, create a new relation with a candidate key of R being the schema, and add it to the result D. This step assures losslessness.

There is only one candidate key of R: AD. Since none of R1, R2 and R3 contains AD, create the relation

R4(A,D) with no FD: {}

Step 4. Simplify the decomposition D by removing relations that are redundant (i.e. that its schema is a subset of the schema of another relation).

No action as there is no redundant relation.

The result relations are all in BCNF.

***Example:***

Consider R(A,B,C,D,E) with {A->BCD, BC->D, D->C}

Using the algorithm,

(1) Canonical cover: {A->BC, BC->D, D->C}; A->D is removed since it is a redundant FD.

(2) The following relations are created:

R1(A,B,C) with {A-> BC},  
R2(B,C,D) with {BC->D, D->C},  
R3(C,D) with {D->C}

(3) There is only one candidate key AE. Since it is not in any of R1, R2 or R3, R4 is created.

R4(A,E)

(4) R3(C,D) is removed as redundant.

As in result, we have:

R1(A,B,C) with {A-> BC}, in BCNF  
R2(B,C,D) with {BC->D, D->C}, in 3NF but not in BCNF  
R4(A,E) with {}, in BCNF

* There are other decomposition algorithms.
* Sometimes, it is not possible to decompose a relation into two relations losslessly and preserve all FD, just to achieve BCNF.

***Example:***

Consider the relation R(A, B, C) with A -> B and C -> B.

R is not in 2NF.  It is not possible to decompose R into *two* relations losslessly while preserving all functional dependencies.

However, it is possible to decompose into *three* BCNF relations losslessly and with all functional dependencies preserved:

R1(A, B),  
R2(B, C) and  
R3(A, C).

Consider the relation R(A, B, C) with A -> B and BC -> A.

R is not in BCNF.  It is not possible to decompose R into BCNF relations losslessly while preserving all FD.